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Research Article

SOME HIGHER ORDER DIFFERENCE DOUBLE SEQUENCE SPACES DEFINED BY AN ORLICZ FUNCTION

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ABSTRACT

In this article we introduce some kth order difference operator on some double sequences operated by an Orlicz function. We introduce some sequence spaces and study different properties of these spaces like completeness, solidity, symmetricity etc. We establish some inclusion results among them.

Keywords: *Orlicz function, difference space, completeness, solid space, symmetric space etc..*

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1. INTRODUCTION

Throughout, a double sequence is denoted by $A = \langle a_{ij} \rangle$. A double sequence is a double infinite array of elements $a_{ij} \in R$ for all $i, j \in N$ and $_2w$ will denote the class of all double sequences.

The initial works on double sequences is found in Bromwich [2]. Later on it was studied by Hardy [6], Moricz [12], Moricz and Rhoades [13], Tripathy [16], Tripathy and Sarma [17], Tripathy, Choudhury and Sarma [18], Basarir and Sonalcan [1] and many others. Hardy [6] introduced the notion of regular convergence for double sequences.

The concept of paranormed sequences was studied by Nakano [14] and Simmons [15] at the initial stage. Later on it was studied by many others.

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [8] as follows:

$$Z(\Delta) = \{ (x_k) \in w : (\Delta x_k) \in Z \}$$

for $Z = c, c_0$ and l_∞, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in N$.

The above spaces are Banach spaces, normed by

$$\| (x_k) \| = | x_1 | + \sup_{k \geq 1} | \Delta x_k |$$

The notion was further investigated by many workers on sequence spaces.

2. DEFINITIONS AND PRELIMINARIES

An *Orlicz function M* is a mapping $M : [0, \infty) \to [0, \infty)$ such that it is *continuous, non-decreasing* and *convex* with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$.

Lindenstrauss and Tzafriri [10] used the idea of Orlicz function to construct the sequence space

$$l^M = \left\{ (x_k) : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right), \text{ for some } \rho > 0 \right\},$$

which is a Banach space normed by

$$\| (x_k) \| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right) \leq 1 \right\}.$$

The space λ^M is closely related to the space λ^p, which is an Orlicz sequence space with $M(x) = |x|^p$, for $1 \leq p < \infty$.

An Orlicz function M is said to satisfy the Δ_2 -condition for all values of u, if there exists a constant $K > 0$, such that $M(2u) \leq K(Mu)$, $u \geq 0$.

Remark 1. Let $0 < \lambda < 1$, then $M(\lambda x) \leq \lambda M(x)$, for all $x \geq 0$.

Let $p = (p_k)$ be a positive sequence of real numbers. If $0 < p_k \leq \sup p_k = H$ and $D = \max(1, 2^{H-1})$, then for a_k , $b_k \in C$ for all $k \in N$, we have

$$| a_k + b_k |^{p_k} \leq D \left\{ | a_k |^{p_k} + | b_k |^{p_k} \right\}.$$

Definition 2.1. A double sequence space E is said to be *solid* if $<\alpha_{ij} a_{ij}> \in E$ whenever $<a_{ij}> \in E$ for all double sequences $<\alpha_{ij}>$ of scalars with $|\alpha_{ij}| \leq 1$ for all $i, j \in N$.

Definition 2.2. Let $K = \{(n_i, k_i) : i \in N ; \ n_1 < n_2 < n_3 < \dots$ and $k_1 < k_2 < k_3 < \dots \}$

$\subseteq N \times N$ and E be a double sequence space. A *K-step space* of E is a sequence space $\lambda_K^E = \{ < a_{n_i k_i} > \in {}_2w : < a_{nk} > \in E \}$.

A *canonical pre-image* of a sequence $< a_{n_i k_i} > \in E$ is a sequence $<b_{nk}> \in E$ defined as follows:

$$b_{nk} = \begin{cases} a_{nk}, & \text{if } (n,k) \in K, \\ 0, & \text{otherwise.} \end{cases}$$

A *canonical pre-image of a step space* λ_K^E is a set of canonical pre-images of all elements in λ_K^E.

Definition 2.3. A double sequence space E is said to be *monotone* if it contains the canonical pre-images of all its step spaces.

Remark 2. From the above notions, it follows that “If a sequence space E solid then E is monotone”.

Definition 2.4. A double sequence space E is said to be *symmetric* if $<a_{ij}>\in E$ implies $<a_{\pi(i)\pi(j)}> \in E$, where π is the permutation of N.

Let M be an Orlicz function and $p = <p_{ij}>$ be a double sequence of strictly positive real numbers. We introduce the following sequence spaces.

$$ {}_2W(M, \Delta^k, p) = \left\{ < a_{ij} > \in {}_2w : \lim_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M\left(\frac{|\Delta^k a_{ij} - L|}{\rho} \right) \right)^{p_{ij}} = 0, \right. $$

$$ \left. \text{for some } \rho > 0 \text{ and } L. \right\} $$

$$ {}_2W_0(M, \Delta^k, p) = \left\{ < a_{ij} > \in {}_2w : \lim_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M\left(\frac{|\Delta^k a_{ij}|}{\rho} \right) \right)^{p_{ij}} = 0, \text{ for some } \rho > 0. \right\} $$

$$ {}_2W_\infty(M, \Delta^k, p) = \left\{ < a_{ij} > \in {}_2w : \sup_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M\left(\frac{|\Delta^k a_{ij}|}{\rho} \right) \right)^{p_{ij}} < \infty, \text{ for some } \rho > 0. \right\} $$

3. RESULTS

Theorem 3.1. *The sequence spaces ${}_2W(M,\Delta^k, p)$, ${}_2W_0(M,\Delta^k, p)$ and ${}_2W_\infty(M,\Delta^k, p)$ are paranormed sequence spaces paranormed by*

$$ g(<a_{ij}>) = \inf \left\{ \rho^{\frac{p_{ij}}{J}} > 0 : \sup_{i,j} \left(M\left(\frac{|\Delta^k a_{ij}|}{\rho} \right) \right) \le 1 \right\}, $$

where $J = \max(1, H)$.

Proof. Clearly $g(0) = 0$, $g(- <a_{ij}>) = g(<a_{ij}>)$. Let $< a_{ij} >, < b_{ij} > \in {}_2W_\infty(M, \Delta^k, p)$. Then there exists some $\rho_1, \rho_2 > 0$ such that

$$ \sup_{i,j} \left(M\left(\frac{|\Delta^k a_{ij}|}{\rho_1} \right) \right) \le 1 $$

$$ \text{and} \quad \sup_{i,j} \left(M\left(\frac{|\Delta^k b_{ij}|}{\rho_2} \right) \right) \le 1. $$

Let $\rho = \rho_1 + \rho_2$. Then we have,

$$ \sup_{m,n} \left(M\left(\frac{|\Delta^k a_{ij} + \Delta^k b_{ij}|}{\rho} \right) \right) \le \frac{\rho_1}{\rho_1+\rho_2} \sup_{i,j} \left(M\left(\frac{|\Delta^k a_{ij}|}{\rho_2} \right) \right) + \frac{\rho_2}{\rho_1+\rho_2} \sup_{i,j} \left(M\left(\frac{|\Delta^k b_{ij}|}{\rho_2} \right) \right) \le 1 $$

Now

$$g(<a_{ij}>+<b_{ij}>) = \inf\left\{(\rho_1+\rho_2)^{\frac{p_{ij}}{J}} > 0 : \sup_{i,j}\left(M\left(\frac{|\Delta^k a_{ij}+\Delta^k b_{ij}|}{\rho_1+\rho_2}\right)\right)\le 1\right\}$$

$$\le \inf\left\{\rho_1^{\frac{p_{ij}}{J}} : \sup_{i,j}\left(M\left(\frac{|\Delta^k a_{ij}|}{\rho_1}\right)\right)\le 1\right\} + \inf\left\{\rho_2^{\frac{p_{ij}}{J}} : \sup_{m,n}\left(M\left(\frac{|\Delta^k b_{ij}|}{\rho_2}\right)\right)\le 1\right\}$$

$$= g(<a_{ij}>) + g(<b_{ij}>)$$

Let $\eta \in C$, then the continuity of the product follows from the following equality.

$$g(\eta<a_{ij}>) = \inf\left\{\rho^{\frac{p_{ij}}{J}} : \sup_{i,j}\left(M\left(\frac{|\Delta^k \eta a_{ij}|}{\rho}\right)\right)\le 1,\ \rho>0\right\}$$

$$= \inf\left\{(|\eta| r)^{\frac{p_{ij}}{J}} : \sup_{m,n}\left(M\left(\frac{|\Delta^k a_{ij}|}{r}\right)\right)\le 1,\ r>0\right\},$$

where $\frac{1}{r} = \frac{|\eta|}{\rho}$.

Proposition 3.2. (*i*) ${}_2W(M, \Delta^k, p) \subset {}_2W_\infty(M, \Delta^k, p)$ (*ii*) ${}_2W_0(M, \Delta^k, p) \subset {}_2W_\infty(M, \Delta^k, p)$. *The inclusions are strict*.

Theorem 3.3. *If* $\sup \frac{p_{ij}}{p_{uv}} < \infty$ *for all* $i \ge u, j \ge v$, *then* ${}_2W(M, \Delta^{k-1}, p) \subset {}_2W_0(M, \Delta^k, p)$. *The inclusion is strict*.

Proof. Let $<a_{ij}> \in {}_2W(M, \Delta^{k-1}, p)$. Then

$$\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^{k-1}a_{ij}-L|}{\rho}\right)\right)^{p_{ij}} = 0, \text{ for some } \rho>0 \text{ and } L. \quad \ldots (2)$$

Since $\sup \frac{p_{ij}}{p_{uv}} < \infty$ so there exists $K > 0$ such that $p_{ij} < K.p_{uv}$ for all $i \ge u, j \ge v$.

Thus from (2) we have, $\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^{k-1}a_{ij}-L|}{\rho}\right)\right)^{p_{i,j+1}} = 0,$

$$\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^{k-1}a_{ij}-L|}{\rho}\right)\right)^{p_{i+1,j}} = 0 \text{ and}$$

$$\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^{k-1}a_{ij}-L|}{\rho}\right)\right)^{p_{i+1,j+1}} = 0.$$

Now for $|\Delta^k a_{ij}| = |\Delta^{k-1}(a_{ij} - a_{i,j+1} - a_{i+1,j} + a_{i+1,j+1})|$

$$= |\Delta^{k-1}a_{ij} - \Delta^{k-1}a_{i,j+1} - \Delta^{k-1}a_{i+1,j} + \Delta^{k-1}a_{i+1,j+1} + L - L + L - L|$$

we have,

$$\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^k a_{ij}|}{\rho}\right)\right)^{p_{ij}}$$

$$\leq \lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^{k-1}a_{ij}-L|}{\rho}+\frac{|\Delta^{k-1}a_{i+1,j}-L|}{\rho}+\frac{|\Delta^{k-1}a_{i,j+1}-L|}{\rho}+\frac{|\Delta^{k-1}a_{i+1,j+1}-L|}{\rho}\right)\right)^{p_{ij}}$$

$$\leq D^2.\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\left(M\left(\frac{|\Delta^{k-1}a_{ij}-L|}{\rho}\right)\right)^{p_{ij}}+\left(M\left(\frac{|\Delta^{k-1}a_{i+1,j}-L|}{\rho}\right)\right)^{p_{ij}}+\left(M\left(\frac{|\Delta^{k-1}a_{i,j+1}-L|}{\rho}\right)\right)^{p_{ij}}+\left(M\left(\frac{|\Delta^{k-1}a_{i+1,j+1}-L|}{\rho}\right)\right)^{p_{ij}}\right\}.$$

$$\leq D^2.\lim_{m,n}\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\left(M\left(\frac{|\Delta^{k-1}a_{ij}-L|}{\rho}\right)\right)^{p_{ij}}+\left(M\left(\frac{|\Delta^{k-1}a_{i+1,j}-L|}{\rho}\right)\right)^{p_{i+1,j}}+\left(M\left(\frac{|\Delta^{k-1}a_{i,j+1}-L|}{\rho}\right)\right)^{p_{i,j+1}}+\left(M\left(\frac{|\Delta^{k-1}a_{i+1,j+1}-L|}{\rho}\right)\right)^{p_{i+1,j+1}}\right\}$$

.

$= 0.$

Thus $<a_{ij}> \in {}_2W_0(M, \Delta^k, p)$ and hence ${}_2W(M, p) \subseteq {}_2W_0(M, \Delta^k, p)$.

The inclusion is strict follows from the following example.

Theorem 3.4. (*i*) *If* $0 < \inf p_{ij} \leq p_{ij} < 1$, *then* ${}_2W(M, \Delta^k, p) \subseteq {}_2W(M, \Delta^k)$.

(*ii*) *If* $1 \leq p_{ij} < \sup p_{ij} < \infty$, *then* ${}_2W(M, \Delta^k) \subseteq {}_2W(M, \Delta^k, p)$.

Proof. The first part of the result follows from the inequality

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^k a_{ij}-L|}{\rho}\right)\right)\leq \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^k a_{ij}-L|}{\rho}\right)\right)^{p_{ij}}$$

and the second part of the result follows from the inequality

$$\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^k a_{ij}-L|}{\rho}\right)\right)^{p_{ij}}\leq \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(M\left(\frac{|\Delta^k a_{ij}-L|}{\rho}\right)\right)$$

Theorem 3.5. *Let* M_1 *and* M_2 *be two Orlicz functions*. *Then*

$${}_2W(M_1, \Delta^k, p) \cap {}_2W(M_2, \Delta^k, q) \subseteq {}_2W(M_1 + M_2, \Delta^k, q).$$

Proof. Let $<a_{ij}> \in {}_2W(M_1, \Delta^k, p) \cap {}_2W(M_2, \Delta^k, q)$. Then

$$\lim_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M_1 \left(\frac{|\Delta^k a_{ij} - L|}{\rho_1} \right) \right)^{p_{ij}} = 0, \quad \text{for some } \rho_1 > 0.$$

$$\lim_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M_2 \left(\frac{|\Delta^k a_{ij} - L|}{\rho_2} \right) \right)^{p_{ij}} = 0, \quad \text{for some } \rho_2 > 0.$$

Let $\rho = \max \{ \rho_1, \rho_2 \}$. The result follows from the following inequality.

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left((M_1 + M_2) \left(\frac{|\Delta^k a_{ij} - L|}{\rho} \right) \right)^{p_{ij}} \leq$$

$$D \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M_1 \left(\frac{|\Delta^k a_{ij} - L|}{\rho_1} \right) \right)^{p_{ij}} + \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M_2 \left(\frac{|\Delta^k a_{ij} - L|}{\rho_2} \right) \right)^{p_{ij}} \right\}.$$

Theorem 3.6. *The sequence space* ${}_2W_\infty(M, \Delta^m, p)$ *is solid and hence monotone*.

Proof. Let $< a_{ij} > \in {}_2W_\infty(M, \Delta^k, p)$ and $< \alpha_{ij} >$ be a scalar sequence such that $|\alpha_{ij}| \leq 1$ for all $i, j \in N$.

Now $M \left(\frac{|\alpha_{ij} \Delta^k a_{ij}|}{\rho} \right) \leq M \left(\frac{|\Delta^k a_{ij}|}{\rho} \right)$

$$\Rightarrow \left(M \left(\frac{|\alpha_{ij} \Delta^k a_{ij}|}{\rho} \right) \right)^{p_{ij}} \leq \left(M \left(\frac{|\Delta^k a_{ij}|}{\rho} \right) \right)^{p_{ij}}$$

$$\Rightarrow \sup_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M \left(\frac{|\alpha_{ij} \Delta^k a_{ij}|}{\rho} \right) \right)^{p_{ij}} \leq \sup_{m,n} \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(M \left(\frac{|\Delta^k a_{ij}|}{\rho} \right) \right)^{p_{ij}} < \infty.$$

Result 3.7. *The sequence spaces* ${}_2W(M, \Delta^k, p)$ *and* ${}_2W_0(M, \Delta^k, p)$ *are not monotone and hence are not solid*.

Proof. The result follows from the following example.

Example 3.1. Let $M(x) = x^p$, $p \geq 1$. Then the double sequence $< a_{ij} >$ defined by $a_{ij} = 1$ for all $i, j \in N$ belongs to ${}_2W(M, \Delta^k, p)$ and ${}_2W_0(M, \Delta^k, p)$. Consider its pre-image $< b_{ij} >$ defined as

$$b_{ij} = \begin{cases} a_{ij}, & \text{if } i + j \text{ is odd.} \\ 0, & \text{otherwise.} \end{cases}$$

Then $< b_{ij} >$ belongs neither to ${}_2W(M, \Delta^k, p)$ nor to ${}_2W_0(M, \Delta^k, p)$ for any k. Hence the spaces ${}_2W(M, \Delta^k, p)$ and ${}_2W_0(M, \Delta^k, p)$ are not monotone and by Remark 3 these are not solid also.

Result 3.8. *The sequence spaces* ${}_2W(M, \Delta^k, p)$, ${}_2W_0(M, \Delta^k, p)$ *and* ${}_2W_\infty(M, \Delta^k, p)$ *are not symmetric*.

Proof. To prove the results consider the following examples.

Example 3.2. Let $M(x) = x^2$, $k = 2$. Consider the sequence $< a_{ij} >$ defined by

$$a_{ij} = \begin{cases} 1, & \text{if } i \text{ is odd for all } j \in N. \\ -1, & \text{otherwise.} \end{cases}$$

Then $\Delta^2 a_{ij} = 0$ for all $i, j \in N$.

Let $< b_{ij} >$ be a rearrangement of the sequence $< a_{ij} >$ defined by

$$b_{ij} = \begin{cases} -1, & \text{if } i + j \text{ is even.} \\ 1, & \text{otherwise.} \end{cases}$$

Then $$\Delta^2 b_{ij} = \begin{cases} -16, & \text{if } i + j \text{ is even.} \\ 16, & \text{otherwise.} \end{cases}$$

Here $< a_{ij} > \in {}_2W_0(M, \Delta^k, p) \subseteq {}_2W(M, \Delta^k, p)$ but $< b_{ij} > \notin {}_2W(M, \Delta^k, p)$.

Example 3.3. Let $M(x) = x^p$, $p \geq 1$, $k = 2$, $p_{ij} = 2$ for all $i, j \in N$. Consider the sequence $< a_{ij} >$ defined by

$$a_{ij} = \begin{cases} 0, & \text{if } i \text{ is even for all } j \in N. \\ i, & \text{otherwise.} \end{cases}$$

Then $\Delta a_{ij} = 0$ for all $i, j \in N$.

Let $< b_{ij} >$ be a rearrangement of the sequence $< a_{ij} >$ defined by

$$b_{ij} = \begin{cases} 0, & \text{if } i + j \text{ is odd.} \\ i, & \text{otherwise.} \end{cases}$$

Then $$\Delta^2 b_{ij} = \begin{cases} 8i + 8, & \text{if } i + j \text{ is even.} \\ -8i - 8, & \text{otherwise.} \end{cases}$$

Here $< a_{ij} > \in {}_2W_\infty(M, \Delta^k, p)$ but $< b_{ij} > \notin {}_2W_\infty(M, \Delta^k, p)$.

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