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RESEARCH ARTICLE / ARAŞTIRMA MAKALESİ

# Türev ve Kısmi İntegral Yöntemi ile Gaus İntegralinin Belirsiz İntegral Çözümü ile Kuantum Mekaniğinde Dalga Fonksiyonunda Kullanımı ve Dalga Fonksiyonunun Konuma ve Zamana Bağlı Tam Çözümleri

## *Derıvative And Partial Integral Method and The use of Gauss Integral in Undefined Integral Solution With Wave Function in Quantum Mechanics and Complete Solutions of The Wave Function Depending on Location and Time*

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**Öz**

Bu çalışma, $\int e^{-\alpha x^2} dx$, Gauss İntegrali'nin diferansiyel denklem ile çözümü[18,19]; kısmi integral yöntemi kullanılarak x diferansiyel altına alınarak belirsiz integral çözümü[29,30,31] ve Gauss İntegrali diferansiyel altına alınarak belirsiz integral çözümü[33,38,39] hem harmonic seri hem de fonksiyon çözümü bulunmuştur. Gauss İntegralinin belirsiz integral çözümü[38] Kuantum Fiziğinde dalga fonksiyonununda yerine koyarak x konum ve k uzayında değişkeni cinsinden $f(x,\alpha)$ dalga fonksiyonu çözümü[44,45] ve bir boyutlu zamana bağlı Schrödinger denkleminde, dalga fonksiyonu denklemi[66,67] bulunur. Direkt uzayda dalga fonksiyonunu k uzayında yaklaşık değeri kullanılmadan kısmi integral yöntemiyle diferansiyel altına alındığında konuma göre k uzayında genel dalga denklemi[72] ve yaklaşık olarak ise $f(x,\alpha)$ dalga fonksiyonu[78] bulunur. $\int e^{-\alpha k^2} e^{ikx} dk$ ve $f(x,t,\alpha) = \int e^{-\alpha k^2} e^{ikx-iwkt} dk$ konuma ve zamana bağlı $k$ uzayında dalga denklemlerinin çözümü[81,87] tam çözümü ve Taylor Serisi çözümü ile de konum ve zamana göre k uzayında dalga denklemi[85,88] bulunur.

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Bu çözüm yönteminin geliştirilmesinde diferansiyel denklem, kısmi integral, harmonik seriler ve Taylor serisi Gauss İntegralinin belirsiz integral çözümünü ve uzayda konuma ve zamana bağlı dalga fonksiyonunun çözümünü elde etmek için kullanılmıştır. Sonuç olarak bu çözüm yöntemi diğer integrallerin belirsiz integral değerlerinin hesaplanması için bir yöntem olacağı ve kuantumlu yapıya sahip integrallerin yaklaşımsal olarak hesaplanabileceği ortaya konulmuştur.

**Anahtar kelimeler:** Gauss İntegrali, Türev, Kısmi İntegral, Belirsiz İntegral, Dalga Fonksiyonu, Kuantum Fiziği, Schrödinger

**Abstract**

This study, $\int e^{-\alpha x^2}dx$, solution of Gaussian Integral with differential equation [18,19]; By using the partial integral method, the indefinite integral solution by taking x under the differential [29,30,31] and the indefinite integral solution by taking the Gaussian integral under the differential [33,38,39] both harmonic series and function solutions are found. Indefinite integral solution of Gaussian Integral [38] In Quantum Physics, the wave function solution f(x,$\alpha$) in terms of $\alpha$ variable in x position and k space by substituting it in wave function [44,45] and in one-dimensional time dependent Schrödinger equation, wave function equation [66, 67] is found. When the wave function in direct space is differentiated by by the partial integral method without using the approximate value of w(k)[50] in space k, the general wave equation in k space by position [72] and approximately the wave function f(x,$\alpha$) [78] is found. $\int e^{-\alpha k^2}e^{ikx}dk$ and $f(x,t,\alpha) = \int e^{-\alpha k^2}e^{ikx-iwkt}dk$min in space k depending on location and time the exact solution of the wave equations [81,87] and the Taylor Series solution give the wave equation [85,88] in k space according to position and time.

**Keywords:** Gaussian Integral, Derivative, Partial Integral, Indefinite Integral, Wave Function, Quantum Physics. Schrödinger

## 1. INDEFİNITE INTEGRAL SOLUTION OF GAUSS INTEGRAL

## INTRODUCTION

In this article, in the development of the solution method, differential equation, partial integral, harmonic series and Taylor series are used to obtain the indefinite integral solution of the Gaussian Integral and the solution of the wave function dependent on position and time in space. As a result, it has been revealed that this solution method will be a method for calculating the indefinite integral values of other integrals and that integrals with quantized structure can be calculated approximately.

### 1.1 Solution Of The Gauss Integral With A Differential Equation

Let's write the indefinite integral solution of the Gaussian integral as the derivative of the product;

$$\int e^{-x^2}dx = xe^{-x^2}.f_n = g(x).f_n$$

$$g = g(x) = xe^{-x^2} \quad (1)$$

Equation 3 is obtained by taking the derivative in equation 1.

$$e^{-x^2} = g'.f_n + g.f_n'$$

$$g'(x) = e^{-x^2} - 2x^2e^{-x^2} \qquad \frac{d}{dx}\int e \quad (2)$$

$$e^{-x^2} = e^{-x^2}(1-2x^2)f_n + xe^{-x^2}f_n' \Rightarrow \quad (3)$$

$$= e^{-x^2}[(1-2x^2)f_n$$

$$1 = (1\text{-}2\,x^2)\,f_n + xf_n' \quad (4)$$

The derivative of equation (4) is taken again;

$0 = -4x\,f_n + (1\text{-}2x^2)f_n' + f_n' + xf_n''$ (5) Equation (5) is found.

$$xf_n'' + 2(1-x^2)f_n' - 4xf_n = 0$$

$$f_n'' + \frac{2}{x}(1-x^2)f_n' - 4x\,f_n = 0 \quad (5)$$

$$x = 0 \; \textit{is the undefined point.}$$

$$f_n'' + p(x).f_n' + q(x)\,f_n = 0$$

$$\lim_{x\to 0} xp(x) = \lim_{x\to 0} x.\frac{2}{x}(1-x^2) = 2$$

$$\lim_{x\to 0} x^2q(x) = \lim_{x\to 0} x^2.(-4) = 0$$

$x = 0'da\ \ regular\ singular\ point$

$f_n' = \sum_{n=0}^{\infty}(n+s)a_n x^{n+s-1}$

$f_n(\text{x}) = \sum_{n=0}^{\infty} a_n x^{n+s} \qquad a_0 \neq 0$

$$f_n'' = \sum_{n=0}^{\infty}(n+s)(\text{n}+\text{s}-1)\, a_n x^{n+s-2} \qquad (6)$$

Let's substitute it into the equation in (5);

x. $f_n''{}'\ due\ to\ x' \quad +1$

$\sum_{n=0}^{\infty}(n+s)(\text{n}+\text{s}-1)\, a_n x^{n+s-1} + 2\sum_{n=0}^{\infty}(n+s)a_n x^{n+s-1} - 2\sum_{n=0}^{\infty}(n+s)a_n x^{n+s+1} - 4\sum_{n=0}^{\infty} a_n x^{n+s+1}$

$\sum_{n=0}^{\infty}[(n+s)(\text{n}+\text{s}-1)+2(\text{n}+\text{s})]\, a_n x^{n+s-1}$
$2\sum_{n=0}^{\infty}[(n+s)+2]\, a_n x^{n+s+1} = 0$

$\sum_{n=0}^{\infty}(n+s)(\text{n}+\text{s}+1)\, a_n x^{n+s-1} - 2\sum_{n=0}^{\infty}[(n+s)+2]\, a_n x^{n+s+1} = 0$

$n = 0\ için\,;$ s.(s+1) $a_0 x^{s-1} + (s+1)(s+2)a_1 x^s$

$n - 1 = n' + 1$

$\sum_{n=-2}^{\infty}(n+s+2)(n+s+3)a_{n+2}\, x^{n+s+1} - 2\sum_{n=0}^{\infty}(n+s+2)\, a_n x^{n+s+1} = 0$

$$\sum_{n=0}^{\infty}[(n+s+2)(n+s+3)\, a_{n+2} - 2(n+s+2)\, a_n]x^{n+s+1} \qquad (7)$$

$\sum_{n=0}^{\infty}[(n+s+2)(n+s+3)\, a_{n+2} - 2(n+s+2)\, a_n]$

$$\text{s.(s+1)}\, a_0 x^{s-1} + \text{(s+1).(s+2)}\, a_1 x^s \qquad (8)$$

(when parentheses common $a_n'$ ).

s.(s+1) $a_0 = 0 \qquad a_0 \neq 0$

(s+1).(s+2). $a_1 = 0$

(n+s+2)(n+s+3) $a_{n+2}$-2(n+s+2) $a_n = 0 \qquad n \geq 0$

s.(s+1) $a_0$=0 $\qquad a_0 \neq 0$

s=0,-1

$$\text{L(x)} = \text{x.}\frac{d^2}{dx^2} + 2(1-x^2)\frac{d}{dx} - 4x \qquad (9)$$

L(-x) = (-x$\frac{d^2}{dx^2}$ +2(1-$x^2$)(−)$\frac{d}{dx}$ − 4(−$x$)

L(-x) = -[x$\frac{d^2}{dx^2}$ +2(1-$x^2$)$\frac{d}{dx}$ −4$x$]

L(-x) = - L(x)

$a_1 \neq 0 \qquad s = -1$

(n+1).(n+2). $a_{n+2}$-2(n+1) $a_n = 0 \qquad n \geq 0$

$a_{n+2} = \frac{2(n+1)}{(n+1)(n+2)} \cdot a_n \qquad n \geq 0$

$$a_{n+2} = \frac{2}{(n+2)} \cdot a_n \qquad n \geq 0 \qquad (10)$$

$n = 0 \qquad a_2 = a_0$

$n = 2 \qquad a_4 = \frac{2}{4}a_2 \qquad a_4 = \frac{1}{2}a_0$

$n = 4 \qquad a_6 = \frac{2}{6}a_4 \qquad a_6 = \frac{1}{2.3}a_0$

$n = 6 \qquad a_8 = \frac{2}{8}a_6 \qquad a_8 = \frac{1}{2.3.4}a_0$

$\vdots \qquad \vdots \qquad \vdots$

$$\Rightarrow a_{2n} = \frac{1}{n!}\, a_0 \qquad (11)$$

(11) found as a result.

$n = 1 \qquad a_3 = \frac{2}{3}a_1$

$n = 3 \qquad a_5 = \frac{2}{5}a_3 \qquad a_5 = \frac{2.2}{5.3}a_1 = \frac{2^2}{3.5}a_1$

$n = 5 \qquad a_7 = \frac{2}{7}a_5 \qquad a_7 = \frac{2^2}{3.5}\cdot\frac{2}{7}\cdot a_1 = \frac{2^3}{3.5\cdot 7}a_1$

$$a_{2n+1} = \frac{2^n}{3.5.7\ldots(2n+1)} \cdot a_1 \quad is\ found. \qquad (12)$$

$$f_n(x) = \sum_{n=0}^{\infty} a_{2n}x^{2n-1} + \sum_{n=0}^{\infty} a_{2n+1}x^{2n+1-1} \qquad (13)$$

When $a_{2n}$ and $a_{2n+1}$ values in (11) and (12) are written instead;

$$f_n(x) = \sum_{n=1}^{\infty} a_0 \frac{1}{n!}x^{2n-1} + \sum_{n=1}^{\infty}\frac{2^n}{3.5\cdot 7\ldots(2n+1)} \cdot a_1 x^{2n} \qquad (14)$$

$$f_n(x) = a_0 \sum_{n=0}^{\infty} \frac{x^{2n-1}}{n!} + a_1 \sum_{n=0}^{\infty} \frac{2^n x^{2n}}{3.5\cdot 7 \ldots (2n+1)} \quad (15)$$

$$\Rightarrow f_n(x) = \sum_{n=0}^{\infty} \frac{2^{2n} n!}{(2n+1)!} x^{2n} \quad (16)$$

solution of the differential equation,

$$f_n(x) = \sum_{n=0}^{\infty} \frac{n!}{(2n+1)!} . (2x)^{2n} \quad (17)$$

The general solution of the second part of the differential equation is found.

$$\int e^{-x^2} dx = xe^{-x^2}. f_n(x)$$

Let's write the general solution (15) in the indefinite integral solution of the Gaussian integral in equation (1);

$$\int e^{-x^2} dx = xe^{-x^2}. \left[ a_0. \sum_{n=0}^{\infty} \frac{x^{2n-1}}{n!} + a_1. \sum_{n=0}^{\infty} \frac{2^n x^{2n}}{3.5\cdot 7 \ldots (2n+1)} \right] \quad (18)$$

Consider the 2nd term with $a_1$;

$$\int e^{-x^2} dx = xe^{-x^2}. \sum_{n=0}^{\infty} \frac{n!}{(2n+1)!} . (2x)^{2n}$$

$$\int e^{-x^2} dx = x \frac{1}{\sqrt{2x}} e^{-x^2}. \sum_{n=0}^{\infty} \frac{n!}{(2n+1)!} . (2x)^{2n+1} \quad (19)$$

Indefinite integral solution of Gaussian integral; It is found as $f_n(x)$ harmonic series product. And its derivative also manifests itself according to the rule in (3).

**1.2 Indefinite Integral Solution By Taking x Under The Differential Using The Partial Integral Method**

Solving the Gaussian integral by partial integral method;

$$I(x, \alpha) = \int e^{-\alpha x^2} dx \quad (20)$$

Using the partial integral method,

$u = e^{-\alpha x^2}$ $\quad v = x$

$du = -2\alpha x e^{-\alpha x^2} dx$ $\quad dv = dx$

$$I(x, a) = xe^{-\alpha x^2} + 2\alpha \int x^2 e^{-\alpha x^2} dx$$

By applying the partial integral again,

$u = e^{-\alpha x^2}$ $\quad v = \frac{x^3}{3}$ $\quad du = -2\alpha x e^{-\alpha x^2} dx$ $\quad dv = x^2\, dx$

whereas; $2\alpha. [\, e^{-\alpha x^2}. \frac{x^3}{3} + 2\alpha \int \frac{x^4}{3} e^{-\alpha x^2}\, dx]$

$$I(x, \alpha) = xe^{-\alpha x^2} + \frac{2\alpha}{3} x^3 e^{-\alpha x^2} + \frac{4\alpha^2}{3} \int x^4 e^{-\alpha x^2}\, dx$$

$; \frac{4\alpha^2}{3} [\frac{x^5}{5} e^{-\alpha x^2} + \frac{2\alpha}{5} \int x^6 e^{-\alpha x^2} dx]$ $\quad u = e^{-\alpha x^2}$ $\quad v = \frac{x^5}{5}$

$du = -2\alpha x e^{-\alpha x^2} dx$ $\quad dv = x^4\, dx$

$$I(x, \alpha) = xe^{-\alpha x^2} + \frac{2\alpha}{3} x^3 e^{-\alpha x^2} + \frac{4\alpha^2}{3.5} x^5 e^{-\alpha x^2} + \frac{2.4\alpha^3}{3.5} \int x^6 e^{-\alpha x^2}\, dx$$

By applying the partial integral again,

$$\int x^6 e^{-\alpha x^2}\, dx = \frac{x^7}{7} e^{-\alpha x^2} + \frac{2\alpha}{7} \int x^8 e^{-\alpha x^2}\, dx$$

$u = e^{-\alpha x^2}$ $\quad v = \frac{x^7}{7}$

$du = -2\alpha x e^{-\alpha x^2} dx$ $\quad dv = x^6\, dx$

$$I(x, \alpha) = xe^{-\alpha x^2} + \frac{2\alpha}{3} x^3 e^{-\alpha x^2} + \frac{2^2\alpha^2}{3.5} x^5 e^{-\alpha x^2} + \frac{2^3\alpha^3}{3.5\cdot 7} x^7 e^{-\alpha x^2} + \frac{2^4\alpha^4}{3.5\cdot 7} \int x^8 e^{-\alpha x^2}\, dx$$

$$I(x, \alpha) = x. e^{-\alpha x^2} [1 + \frac{2\alpha}{3} x^2 + \frac{2^2\alpha^2}{3.5} x^4 + \frac{2^3\alpha^3}{3\cdot 5.7} x^7 e^{-\alpha x^2} + \frac{2^4\alpha^4}{3.5\cdot 7.9} x^8 + \cdots + \frac{2^n\alpha^n x^{2n}}{3.5\cdot 7.9 \ldots (2n+1)} + \frac{2^q\alpha^q}{3.5\cdot 7.9 \ldots (2q+1)} \int x^{2\alpha} e^{-\alpha x^2}\, dx$$

derivative of $I(x, \alpha)$; It gives itself according to the rule in (3).

$$I(x, \alpha) = x. e^{-\alpha x^2} [1 + \sum_{n=1}^{q} \frac{2^n\alpha^n x^{2n}}{3.5.7 \ldots (2n+1)} + \frac{2^q\alpha^q}{3.5.7 \ldots . (2q+1)} I_q(x, \alpha)] \quad (21)$$

(21) the general equation is found.

Where; $I_q(x, \alpha) = \int x^{2q} e^{-\alpha x^2}\, dx$

$$I_q(x, \alpha) = (-1)^q \frac{d^q I(x, \alpha)}{dx^q} \qquad q = 1,2,3, \ldots . \quad (22)$$

The last term because q is too big;

$$\lim_{q\to\infty} \int x^{2q} e^{-\alpha x^2}\, dx \to \frac{2}{e} \quad (23)$$

(23) It takes the value. here; $e = 2.7182\ldots.$

Thus, by the approximation method, the indefinite integral value;

$$\int e^{-\alpha x^2}\,dx = x.e^{-\alpha x^2}.\left[1+\sum_{n=1}^{q}\frac{2^n\alpha^n x^{2n}}{3.5.7...(2n+1)}+\frac{2^{q+1}\alpha^q}{3.5.7....(2q+1)}.\frac{1}{e}\right] \quad (24)$$

(24) is found as.

$$3.5.7\ldots(2n+1)=\frac{2.3.4.5.....(2n+1)}{2^n(1.2.3.4....n)}=\frac{(2n+1)!}{2^n.n!} \quad (25)$$

$$\frac{1}{3.5.7....(2n+1)}=\frac{2^n.n!}{(2n+1)!} \quad (26) \qquad 3.5.7\ldots(2q+1)=\frac{(2q+1)!}{2^q.q!} \quad (26)$$

$$\lim_{q\to\infty}\frac{2^{2q}.q!}{(2q+1)!}\to 0 \quad (28) \qquad \lim_{q\to\infty}\int x^{2q}\,e^{-\alpha x^2}\,dx\to\frac{2}{e} \quad (28)$$

Where; $0<\alpha\leq 1$ $\qquad e = 2.7182\ ....$

Let's substitute the equations (25), (26), (27) and (28) in equation (24);

$$\int e^{-\alpha x^2}\,dx = xe^{-\alpha x^2}.[1+\sum_{n=1}^{\infty}\frac{2^{2n}\alpha^n x^{2n}.n!}{(2n+1)!}]$$

$$\int e^{-\alpha x^2}\,dx = xe^{-\alpha x^2}.[\sum_{n=0}^{\infty}\frac{2^{2n}\alpha^n x^{2n}.n!}{(2n+1)!}] \quad (29)$$

if we make an arrangement in the equation (29);

$$I(x,\alpha)=\int e^{-\alpha x^2}\,dx = xe^{-\alpha x^2}.[\sum_{n=0}^{\infty}\frac{n!}{(2n+1)!}\left(2\sqrt{\alpha x}\right)^{2n}] \quad (30)$$

Equation number (30) is found.

$$I(x,\alpha)=\int e^{-\alpha x^2}\,dx = xe^{-\alpha x^2}.\sum_{n=0}^{\infty}\frac{2^n n!}{(2n+1)!}\left(\sqrt{2\alpha}x\right)^{2n}]$$

$$I(x,\alpha)=\frac{x\sqrt{2\alpha}}{\sqrt{2\alpha}}e^{-\alpha x^2}.\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\left(\sqrt{2\alpha}x\right)^{2n}]$$

$$I(x,\alpha)=\frac{1}{\sqrt{2\alpha}}e^{-\alpha x^2}.\sum_{n=0}^{\infty}\frac{\left(\sqrt{2\alpha}x\right)^{2n+1}}{(2n+1)}$$

$$I(x,\alpha)=\int e^{-\alpha x^2}\,dx=\frac{1}{\sqrt{2\alpha}}e^{-\alpha x^2}.\operatorname{arctanh}\left(\sqrt{2\alpha}x\right)+c \quad (31)$$

The equation takes its simplest form as follows.

## 1.3 Understanding Integral Solution Of The Gauss Integral By Differentıalizing With Both Harmonic Series And Function Solution

$$\boldsymbol{I(x,\alpha)=\int e^{-\alpha x^2}dx=\int\left(-\frac{1}{2\alpha x}\right)de^{-\alpha x^2}=-\frac{1}{2\alpha x}e^{-\alpha x^2}-\int e^{-\alpha x^2}d\left(-\frac{1}{2\alpha x}\right)}$$

$$I(x,\alpha)=-\frac{1}{2\alpha x}e^{-\alpha x^2}-\int\left(-\frac{1}{2\alpha}\right).\left(-\frac{1}{x^2}\right)e^{-\alpha x^2}dx=-\frac{1}{2\alpha x}e^{-\alpha x^2}-\int\left(\frac{1}{2\alpha x^2}\right).\left(-\frac{1}{2\alpha x}\right)de^{-\alpha x^2}$$

$$I(x,\alpha)=-\frac{1}{2\alpha x}e^{-\alpha x^2}+\int\left(\frac{1}{2^2\alpha^2x^3}\right)de^{-\alpha x^2}=-\frac{1}{2\alpha x}e^{-\alpha x^2}+\frac{1}{2^2\alpha^2x^3}e^{-\alpha x^2}-\int e^{-\alpha x^2}d\left(\frac{1}{2^2\alpha^2x^3}\right)$$

$$I(x,\alpha)=-\frac{1}{2\alpha x}e^{-\alpha x^2}+\frac{1}{2^2\alpha^2x^3}e^{-\alpha x^2}-\int\left(\frac{1.(-3)}{2^2\alpha^2x^4}\right)e^{-\alpha x^2}dx$$

$$I(x,\alpha)=-\frac{1}{2\alpha x}e^{-\alpha x^2}+\frac{1}{2^2\alpha^2x^3}e^{-\alpha x^2}-\frac{3}{2^3\alpha^3x^5}\frac{3.5}{2^4\alpha^4x^7}e^{-\alpha x^2}-\cdots+\int\frac{(-1)^{n+1}.1.3.5...(2n+1)}{2^n\alpha^n x^{2n+1}}e^{-\alpha x^2}dx \quad (32)$$

General equation no (32) and (33) is found.

$$\int e^{-\alpha x^2}\,dx=\frac{1}{2\alpha x}e^{-\alpha x^2}\left[-1+\frac{1}{2\ \alpha\ x^2}-\frac{3}{2^2\alpha^2x^4}+\frac{3.5}{2^3\alpha^3x^6}-\frac{3.5.7}{2^4\alpha^4x^8}+\cdots+\frac{1.3.5.7....(2n+1)}{2^n\alpha^n x^{2n}}\right]$$

$$\int e^{-\alpha x^2}\,dx=\frac{1}{2\alpha x}e^{-\alpha x^2}.\left[\sum_{n=0}^{\infty}\frac{(-1)^{n+1}.1.3.5...(2n+1)}{2^n\alpha^n x^{2n}}\right] \quad (33)$$

Let's write the equations (25), (26), (27) and (28) in equation (33);

$$\int e^{-\alpha x^2}\,dx=\frac{1}{2\alpha x}e^{-\alpha x^2}.[-\sum_{n=0}^{\infty}\frac{(-1)^n.(2n+1)!}{2^n.\alpha^n.x^{2n}.(2n)!}]$$

$$\int e^{-\alpha x^2}\,dx=\frac{e^{-\alpha x^2}}{\sqrt{2\alpha}}[\sum_{n=0}^{\infty}\frac{(-1)^n.(2n+1).(2n)!}{(2n)!.(\sqrt{2\alpha}x)^{2n+1}}]$$

$$\int e^{-\alpha x^2}\,dx=\frac{e^{-\alpha x^2}}{\sqrt{2\alpha}}.[-\sum_{n=0}^{\infty}(-1)^n.(2n+1).\left(\frac{1}{\sqrt{2\alpha}x}\right)^{2n+1}]$$

If we substitute $i^2$ for -1, $(-1)^n=i^{2n}$ and i.i in place of -1 in front of the sum;

$$\int e^{-\alpha x^2}\,dx=\frac{e^{-\alpha x^2}}{\sqrt{2\alpha}}[\sum_{n=0}^{\infty}\frac{i[(i)^{2n}].i}{\frac{1}{(2n+1)}}.\left(\frac{1}{\sqrt{2\alpha}x}\right)^{2n+1}]$$

$$\int e^{-\alpha x^2}\,dx=\frac{e^{-\alpha x^2}}{\sqrt{2\alpha}}.[i.\sum_{n=0}^{\infty}\frac{1}{\frac{1}{(2n+1)}}.\left(\frac{i}{\sqrt{2\alpha}x}\right)^{2n+1}] \quad (34)$$

$$\operatorname{arctanh}(x)=\operatorname{arccoth}\left(\frac{1}{x}\right) \quad (35)$$

since (35);

$$arctanhx=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)}=\sum_{n=0}^{\infty}\frac{(2n+1)}{(-i\sqrt{2\alpha}x)^{2n+1}}$$

$$\operatorname{arctanh}(-ix)=-i.arctanhx$$

becomes;

$$\int e^{-\alpha x^2}\,dx = \frac{1}{\sqrt{2\alpha}} e^{-\alpha x^2}.i\left[\operatorname{arctanh}\left(-i\sqrt{2\alpha x}\right)\right] + c \tag{37}$$

In general, the function value of the Gaussian integral;

$$\int e^{-\alpha x^2}\,dx = \frac{1}{\sqrt{2\alpha}} e^{-\alpha x^2}.\operatorname{arctanh}\left(\sqrt{2\alpha x}\right) + c \tag{38}$$

Logarithmically inverse function;

arctanh (x)=$\frac{1}{2}\ln\left[\frac{1+x}{1-x}\right]$ ; $|x| < 1$

equal;

$$\int e^{-\alpha x^2}\,dx = \frac{1}{2\sqrt{2\alpha}} e^{-\alpha x^2}.\ln\left[\frac{1+\sqrt{2\alpha x}}{1-\sqrt{2\alpha x}}\right] + c \quad \left|\sqrt{2\alpha x}\right| < 1 \tag{39}$$

The indefinite integral of the Gaussian Integral; In both the differential equation and the partial integral method, the variables were determined differently and the same value was found in all three solutions.

**2.1 Solutıon Of The Indefınite Integral Solution Of Gauss Integral [38] In Quantum Physıcs[1] Wave Function In x Position And k Space f(X,$\alpha$) Wave Function As Variable f(x,)**

Wave packets in Quantum Physics and the use of Gauss Integral in indefinite integral solution in uncertainty connections;

Wave function in space;

$$f(x) = \int_{-\infty}^{\infty} g(k).\,e^{ikx}\,dk \tag{40}$$

Let's consider the defined function. The real part of f(x); It is $\int_{-\infty}^{\infty} g(k).coskxdk$. And this is linear overlapping of waves of wavelength λ=2π/k. . Because the wave reproduces itself when x is changed to x+2π/k for each value of k.[1]

$$g(k) = e^{-\alpha(k-k_0)^2} \tag{41}$$

Let's choose (41).

Then the derivative can be taken.

($k' = k - k_0$ including,

$$f(x) = \int_{-\infty}^{\infty} g(k).\,e^{i(k-k_0)x}.\,e^{ik_0x}dk$$

$$f(x) = e^{ik_0x}\int_{-\infty}^{\infty} e^{ik'x}e^{-\alpha k'^2}dk'$$

$$f(x) = e^{ik_0x}\int_{-\infty}^{\infty} e^{-\alpha[k'-(ix/2\alpha)]^2}\,e^{-(x^2/4\alpha)}\,dk'$$

[1] This is;

$$k' - (ix/2\alpha) = q$$

to write and made to leave the derivative along the real axis.

$\int_{-\infty}^{\infty} e^{-\alpha k^2}\,dk = \sqrt{\frac{\pi}{\alpha}}$ [1] is used;

$$\text{f(x)}=\sqrt{\frac{\pi}{\alpha}}\,e^{ik_0x}e^{-(x^2/4\alpha)} \tag{42}$$

(42) is obtained.

When $\left|e^{ik_0x}\right|^2 = 1$,

$e^{ik_0x}$ product is known as a'phase multipler'.

$$|f(x)|^2 = \frac{\pi}{\alpha}e^{-x^2/2\alpha}\ will\ be.\ [1] \tag{43}$$

If we substitute the indefinite integral value in the formula of the Gaussian integral (38);

When used as $\int e^{-\alpha k^2}dk = \frac{1}{\sqrt{2\alpha}}e^{-\alpha k^2}.\operatorname{arctanh}\left(\sqrt{2\alpha k}\right)$

$$f(x,\alpha) = \frac{1}{\sqrt{2\alpha}}e^{-\alpha k^2}.\operatorname{arctanh}\left(\sqrt{2\alpha k}\right).e^{ik_0x}.\,e^{-(x^2/4\alpha)} \tag{44}$$

$$|f(x,\alpha)|^2 = \frac{1}{2\alpha}e^{-2\alpha k^2}.\operatorname{arctanh}^2\left(\sqrt{2\alpha k}\right).\,e^{-(x^2/2\alpha)} \tag{45}$$

**2.2 Wave Functıon Equatıon In The One-Dimensional, Time-Dependent Schrödinger Equation (66,67)**

Since it has a spatial variation only in the x direction but not in the y and z directions, if we add a simple plane wave time factor;

It can be written as $e^{ikx-iwt}$

*Where* $w = 2\pi\nu$ *is the angular frequency.*

Since the quantity k depends on the wavelength by $k = 2\pi/\lambda$), this can be written in a simple wave form $e^{2\pi i[(x/\lambda)-\nu t]}$. (46)

Here, with the simple relation $\nu$=c/$\lambda$

$e^{2\pi i(x-ct)/\lambda} = e^{ik(x-ct)}$ becomes the formula. (47)

f(x,t) $= \int_{-\infty}^{\infty} g(k) \,.\, e^{ik(x-ct)}$ dk = f(x-ct)

f(x,t) = $\int g(k).\, e^{ikx-iwkt} dk$ (48)

f(x,t) = $\int g(k).\, e^{ikx-iwkt} dk$ (49)

We can write;

$w(k) \approx w(k_0) + (k-k_0)\left(\frac{dw}{d_k}\right)_{k_0} + \frac{1}{2}(k-k_0)^2\left(\frac{d^2\omega}{dk^2}\right)_{k_0}$ (50)

f(x,t)= $e^{ik_0x}e^{-i\omega(k_0)t}\int e^{-\alpha k'^2}.$
$e^{-i\left(k'^2/2\right).[(d^2w/dk^2)].t}.\, e^{ik'[x-(\frac{dw}{dk}).t]}dk'$[1] (51)

Leaving aside the previous phase multiplication, the form of the x and t coordinates can be written as the propagation speed of the packet, that is, the group speed, as follows;

Where $v_g = \left(\frac{dw}{d_k}\right)_{k_0}$ Like this; (52)

İf $\frac{1}{2}\left(\frac{d^2w}{dk^2}\right)_{k_0} = \beta$ is defined; (53)

We can write the formula;

$f(x,t) = e^{i[k_0x-w(k_0)t]}\int e^{ik'(x-vgt)}.\, e^{-(\alpha+i\beta t)k'^2}\, dk$ (54)

If x-$v_g$t is substituted for x and $\alpha + i\beta\, t$ is substituted for $\alpha$

$f(x,t) = e^{i[k_0x-w(k_0)t]}.\left(\frac{\pi}{\alpha+i\beta t}\right)^{1/2}.\, e^{-[\frac{(x-Vgt)^2}{4(\alpha+i\beta t)}]}$ (55)

formula is obtained.

The most important result is: If it represents a particle with momentum p and kinetic energy $P^2/2m$

It should be $v_g = \frac{dw}{dk} = \frac{\rho}{m}$ (56)

$|f(x,t)|^2 = \left(\frac{\pi}{\alpha^2+\beta^2t^2}\right)^{1/2}.\, e^{-[\frac{\alpha(x-v_gt)^2}{2(\alpha^2+\beta^2t^2)}]}$ (57)

E=ħ$w$ (58)

by establishing the connection $w = \frac{p^2}{2m\hbar}$ (59)

To be consistent, $k = \frac{2\pi}{\lambda} = \frac{p}{\hbar}$ (60)

This connection needs to be made between them.

Let's substitute the solution of the Gaussian Integral (38);

$f(x,t) =$

$\frac{1}{\sqrt{2(\alpha+i\beta t)}}e^{-(\alpha+i\beta t)k^2}.\mathrm{arctanh}[\sqrt{2(\alpha+i\beta t)}.k]\, e^{-[\frac{(x-v_gt)^2}{4(\alpha+i\beta t)}]}$ (61)

In quantum physics, by solving the Gauss Integral, the position-time equation of the wave function is found as the function value.[1]

On the other hand, our wave equation is;

Itcanbewrittenas; $\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}.\int dp\emptyset(p).\in^{\frac{i(px-Et)}{\hbar}}$

$i\hbar\frac{\partial\Psi(x,t)}{dt} = \frac{1}{\sqrt{2\pi\hbar}}.\int dp\emptyset(p).E.\in^{\frac{i(px-Et)}{\hbar}}$

$= \frac{1}{\sqrt{2\pi\hbar}}.\int dp\emptyset(p)\frac{p^2}{2m}\in^{\frac{i(px-Et)}{\hbar}}$

$= -\frac{\hbar}{2m}.\frac{\partial^2\psi(x,t)}{dx^2}$ become.[1] (62)

The wave function is a general solution of the differential equation.

One of the solutions of the wave packet is located between the widths in x and k spaces;

$\Delta k.\Delta x \geq 1$ is the reverse reciprocity connection. (63)

If we take the derivative $\hbar k = p \Rightarrow \hbar\partial k = \partial p$

*that is;* $\Delta k = \frac{\Delta p}{\hbar}$

If we write the value $\Delta p.\Delta x \geq \hbar$ instead;

$\boldsymbol{\Delta p.\Delta x} \geq \hbar$ [1]

One-dimensional time dependent Schrodinger equation;

Here the wave function is;

$i\hbar\frac{\partial\Psi(x,t)}{dt} = -\frac{\hbar}{2m}.\frac{\partial^2\psi(x,t)}{dx^2} + V.\psi(x,t)$

$\psi(x,t) =$ (65)

$\frac{1}{\sqrt{2(\alpha+i\beta t)}}e^{-(\alpha+i\beta t)k^2}.\mathrm{arctanh}[\sqrt{2(\alpha+i\beta t)}.k].\, e^{-[\frac{(x-v_gt)^2}{4(\alpha+i\beta t)}]}$ (66)

$\psi(x,t) = \frac{1}{2\sqrt{2(\alpha+i\beta t)}}e^{-(\alpha+i\beta t)k^2}.\ln\left[\frac{1+\sqrt{2(\alpha+i\beta t)}k}{1-\sqrt{2(\alpha+i\beta t)}k}\right].\, e^{-[\frac{(x-v_gt)^2}{4(\alpha+i\beta t)}]}$ (67)

$\Delta p.\Delta x \geq \frac{\hbar}{2}$ is found. (68)

$g(k) = e^{-\alpha k^2}$ (69)

$f(x) = \int g(k)\, e^{ikx}dk$ [1] (70)

### 2.3 Direct Space Wave Functıon In Space K *w (k)*, By The Method Of Partial Integratıon Wıthout Approxımate Value $e^{ikx}$, Differential, General Wave In Space k (72)

Let's write the wave function in k space using the equations (69) and (70);

$$f(x) = \int g(k)\, e^{ikx} dk = \int e^{-\alpha k^2} e^{ikx} dk =$$
$$\int e^{-\alpha k^2} \left(\frac{1}{ix}\right) de^{ikx} = \frac{1}{ix}\left[e^{-\alpha k^2}.e^{ikx} - \int e^{ikx}\, de^{-\alpha k^2}\right]$$

$\vdots \qquad \vdots \qquad \vdots$

In the last equation, the integral value of the function, equation (70), let's take it to the common denominator and apply partial integration continuously;

$$(1+\frac{2\alpha}{i^2x^2}).\int e^{-\alpha k^2} e^{ikx} dk = \frac{1}{ix}\Big[e^{-\alpha k^2}.e^{ikx} + \frac{2\alpha}{ix} k e^{-\alpha k^2}.e^{ikx} + \frac{2^2\alpha^2}{ix}\int k^2 e^{-\alpha k^2} e^{ikx} dk\Big] \tag{71}$$

$\vdots \qquad \vdots \qquad \vdots$

$$\left(1+\frac{2\alpha}{ix}+\frac{1}{\alpha}.\frac{2^3\alpha^3}{i^3x^3}+5.\frac{2^5\alpha^5}{i^5x^5}+ \cdots b_{n-2}.\frac{2^{n-2}\alpha^{n-2}}{i^{n-2}x^{n-2}}\right).\int e^{-\alpha k^2} e^{ikx} dk = \Bigg[e^{-\alpha k^2}.e^{ikx} + \frac{2\alpha}{ix} k e^{-\alpha k^2}.e^{ikx} + \frac{2^2\alpha^2}{i^2x^2} k^2 e^{-\alpha k^2}.e^{ikx} + \frac{2^3\alpha^3}{i^3x^3}(\frac{2}{\alpha}k - k^3).e^{-\alpha k^2}.e^{ikx} + \frac{2^4\alpha^4}{i^4x^4}\left(\frac{5}{\alpha}k^2 - k^4\right).e^{-\alpha k^2}.e^{ikx} + \frac{2^5\alpha^5}{i^5x^5}\left(\frac{5}{\alpha}k + \frac{1}{\alpha}k^3 + k^5\right).e^{-\alpha k^2}.e^{ikx} + + \frac{2^6\alpha^6}{i^6x^6}\left(-\frac{3}{2\alpha^2}k^2 + \frac{5}{2\alpha}k^2 + \frac{1}{2\alpha}k^3 - \frac{5}{2\alpha}k^4 + k^6\right).e^{-\alpha k^2}.e^{ikx} + \frac{2^7\alpha^7}{i^7x^7}\int \Big[\left(\frac{3}{\alpha^2} - \frac{5}{2\alpha}\right).k + \left(\frac{5}{2\alpha} - \frac{3}{2^2\alpha^2}\right).k^2 - \frac{3}{2\alpha^2}k^3 + \frac{1}{2\alpha}k^4 - \frac{11}{2\alpha}k^5 + k^7\Big].e^{-\alpha k^2} de^{ikx} + \cdots + \frac{2^n\alpha^n}{i^nx^n}\int [a_1.k + a_2.k^2 + a_3k^2 + a_4k^2 + \cdots + a_{n-1}k^{n-1} + a_nk^n].e^{-\alpha k^2} de^{ikx}\Bigg| \tag{72}$$

**As a result,**

the general equation (72) of the $f(x) = \int e^{-\alpha k^2} e^{ikx} dk$ function is found.

### 2.4 Approximately Wave Function (78)

$a_n = a_1, a_2, a_3, \ldots a_{n-1}, a_n = constant$ n=1,2,3,4,…

$b_n = b_1, b_2, b_3, \ldots b_{n-3}, b_{n-2} = constant$ n=1,2,3,4,… (73)

Since;

$$\lim_{n\to\infty} \int k^{2n} e^{-\alpha k^2} dx \to \frac{2}{e} \text{ and } \left|\frac{2\alpha}{ix}\right| < 1 \tag{74}$$

$$\lim_{n\to\infty} \frac{2^n\alpha^n}{i^nx^n} = (\frac{2\alpha}{ix})^n = 0 \tag{75}$$

$$\sum_{n=0}^{\infty} \frac{2^n\alpha^n}{i^nx^n} = \frac{1}{1-\frac{2\alpha}{ix}} = \frac{ix}{ix-2\alpha} \tag{76}$$

$$\lim_{n\to\infty} \frac{2^n\alpha^n}{i^nx^n}\int [a_1.k + a_2.k^2 + a_3k^2 + a_4k^2 + \cdots + a_{n-1}k^{n-1} + a_nk^n].e^{-\alpha k^2} de^{ikx} \to 0 \tag{77}$$

$$\left(1+\frac{2\alpha}{ix}+\frac{1}{\alpha}.\frac{2^3\alpha^3}{i^3x^3}+5.\frac{2^5\alpha^5}{i^5x^5}+\left(\frac{5\alpha^2-6\alpha}{2\alpha^2}\right).\left(\frac{2^7\alpha^7}{i^7x^7}\right)\right).\int e^{-\alpha k^2} e^{ikx} dk = \Big[1 + \frac{2^2\alpha^2}{i^2x^2}k^2 + \frac{2^3\alpha^3}{i^3x^3}.(\frac{2}{\alpha}k - k^3) + \frac{2^4\alpha^4}{i^4x^4}.\left(\frac{5}{\alpha}k^2 - k^4\right) + \frac{2^5\alpha^5}{i^5x^5}.\left(\frac{5}{\alpha}k + \frac{1}{\alpha}k^3 + k^5\right) + \frac{2^6\alpha^6}{i^6x^6}.\left(-\frac{3}{2\alpha^2}k^2 + \frac{5}{2\alpha}k^2 + \frac{1}{2\alpha}k^3 - \frac{5}{2\alpha}k^4 + k^6\right) + \frac{2^7\alpha^7}{i^7x^7}.\left(\left(\frac{3}{\alpha^2} - \frac{5}{2\alpha}\right).k + \left(\frac{5}{2\alpha} - \frac{3}{2^2\alpha^2}\right).k^2 - \frac{3}{2\alpha^2}k^3 + \frac{1}{2\alpha}k^4 - \frac{11}{2\alpha}k^5 + k^7\right)\Big].e^{-\alpha k^2}.e^{ikx} + c \tag{78}$$

Positional wave equation in space; with the approximate method, the function ; the 7th degree equation of x and k values is found by equation (78).

### 2.5 $\int e^{-\alpha k^2} e^{ikx} dk$ *and* $f(x,t,\alpha) = \int e^{-\alpha k^2} e^{ikx - iwkt} dk$ In Location And Time Dependence , Exact Solution Of Wave Equations [81,87]

Approaching it from another angle, let's differentiate the expression $e^{-\alpha k^2}$ in the partial integral;

$$f(x) = \int g(k) e^{ikx} dk = \int e^{-\alpha k^2} e^{ikx} dk =$$
$$\int e^{ikx}.\left(\frac{1}{-2\alpha k}\right) de^{-\alpha k^2} = -\frac{1}{2\alpha}.\left[\frac{1}{k}e^{-\alpha k^2}.e^{ikx} - \int e^{-\alpha k^2} d(\frac{1}{k}e^{ikx})\right]$$

$\vdots \qquad \vdots \qquad \vdots$

$$= -\frac{1}{2\alpha k}e^{-\alpha k^2}.e^{ikx} - \left(\frac{1}{2^2\alpha^2k^2}\right).(-\frac{1}{k} + ix).e^{-\alpha k^2}.e^{ikx} + \frac{1}{2^3\alpha^3k^3}\left(\frac{3}{k^2} - \frac{3\text{ix}}{k} + i^2x^2\right).e^{-\alpha k^2}.e^{ikx} - \frac{1}{2^4\alpha^4}\int \left(-\frac{3.5}{k^6} + \frac{3.4\text{ix}}{k^5} - \frac{3i^2x^2}{k^4} + \frac{3\text{ix}}{k^5} - \frac{3i^2x^2}{k^4} + \frac{i^3x^3}{k^3}\right).(\frac{1}{k})e^{ikx} de^{-\alpha k^2}$$

$\vdots \qquad \vdots \qquad \vdots$

$$\int e^{-\alpha k^2} e^{ikx} dk = -\frac{1}{2\alpha k}e^{-\alpha k^2}.e^{ikx} - \left(\frac{1}{2^2\alpha^2k^2}\right).(-\frac{1}{k} + ix).e^{-\alpha k^2}.e^{ikx} + \frac{1}{2^3\alpha^3k^3}\left(\frac{3}{k^2} - \frac{3\text{ix}}{k} + i^2x^2\right).e^{-\alpha k^2}.e^{ikx} - \frac{1}{2^4\alpha^4k^4}\left(-\frac{3.5}{k^3} + \frac{15\text{ix}}{k^2} - \frac{6i^2x^2}{k} + i^3x^3\right)e^{-\alpha k^2}.e^{ikx} + \frac{1}{2^5\alpha^5k^5}\left(-\frac{3.5}{k^3} + \frac{15\text{ix}}{k^2} - \frac{6i^2x^2}{k} + i^3x^3\right)e^{-\alpha k^2}.e^{ikx} - \frac{1}{2^6\alpha^6k^6}(\frac{3.5.8}{k^4} - \frac{15.8\text{ix}}{k^3} - \frac{17.3i^2x^2}{k} - 5i^3x^3 - \frac{6i^3x^3}{k} + i^4x^4) + \frac{1}{2^5\alpha^5}\int e^{-\alpha k^2}.\left(-\frac{3.5.8.9}{k^{10}} + \frac{15.8^2\text{ix}}{k^9} + \frac{17.3.7i^2x^2}{k^8} + \frac{5^2i^3x^3}{k^5} + \frac{6^2i^3x^3}{k^7} - \frac{5i^4x^4}{k^6} + \frac{3.5.8\text{ix}}{k^9} - \frac{15.8i^2x^2}{k^8} - \frac{17.3i^3x^3}{k^7} - \frac{5i^4x^4}{k^5} - \frac{6i^4x^4}{k^6} + \frac{i^5x^5}{k^5}\right).e^{ikx} dk \tag{79}$$

Let's write equation (79) above in the simplest way, in binomial expansion;

$$\int e^{-\alpha k^2} e^{ikx} dk = e^{-\alpha k^2}.e^{ikx}.\sum_{n=1}^{\infty}[\frac{(-1)^n}{2^n\alpha^n k^n}].\sum_{p=0}^{n-2}\binom{n-2}{p}(\frac{1}{k})^p(ix)^{n-2-p} + c \quad (80)$$

$$f(x) = \int e^{-\alpha k^2} e^{ikx} dk = e^{-\alpha k^2}.e^{ikx}.\sum_{n=1}^{\infty}[\frac{(-1)^n}{2^n\alpha^n k^n}].(\frac{1}{k}+ix)^{n-2} + c \qquad c = constant \quad (81)$$

Since; $\sum_{n=0}^{\infty}(-1)^n(\frac{1}{2\alpha k})^n = \frac{1}{1+2\alpha k} \quad |2\alpha k| < 1$ ; (82)

$$\sum_{n=1}^{\infty}(-1)^n(\frac{1}{2\alpha k})^n = \frac{1}{1+2\alpha k} - 1 = \frac{-2\alpha k}{1+2\alpha k} \quad (83)$$

$$\sum_{n=0}^{\infty}(-1)^n.[(\frac{1}{2\alpha k}).(\frac{1}{k}+ix)]^n = \frac{1}{1+\frac{1}{2\alpha k}.(\frac{1}{k}+ix)} = \frac{2\alpha k^2}{2\alpha k^2+2\alpha ixk+1} \quad (84)$$

(84) is found from the Taylor series.

$$f(x,\alpha) = \int e^{-\alpha k^2} e^{ikx} dk$$

$$\int e^{-\alpha k^2} e^{ikx} dk = e^{-\alpha k^2} e^{ikx}.\sum_{n=1}^{\infty}[\frac{(-1)^n}{2^n\alpha^n k^n}](\frac{1}{k}+ix)^{n-2} + C$$

$$f(x,\alpha) = \int e^{-\alpha k^2} e^{ikx} dk = [\sum_{n=1}^{\infty}(-1)^2.(\frac{1}{2\alpha k})^n.(\frac{1}{k}+ix)^n \;].(\frac{1}{k}+ix)^{-2}.e^{-\alpha k^2}.e^{ikx} + C$$

$$f(x,\alpha) = \int e^{-\alpha k^2} e^{ikx} dk = (\sum_{n=1}^{\infty}(-1)^n[\left(\frac{1}{2\alpha k}\right).\left(\frac{1}{k}+ix\right)]^n \;).(\frac{1}{k}+i\square)^{-2}.e^{-\alpha k^2}.e^{ikx} + C$$

$$\int e^{-\alpha k^2} e^{ikx} dk = -\frac{(2\alpha ikx+1)}{2\alpha k^2+2\alpha ixk+1}.\frac{k^2}{(1+ikx)^2} \;.e^{-\alpha k^2}.e^{ikx} + C \quad (85)$$

The general wave equation in k space is found with equation (85).

Time dependent wave equation;

$$f(x,\alpha,t) = \int g(k).e^{ikx-iwkt} dk \quad (86)$$

$$f(x,\alpha,t) = \int e^{-\alpha k^2} e^{ikx-iwkt} dk = e^{-\alpha k^2}.e^{ikx-iwkt}.\sum_{n=1}^{\infty}[\frac{(-1)^n}{2^n\alpha^n k^n}][\frac{1}{k}+(ix-iwt)]^{n-2} + C$$

(87)

**2.6 Wave Equation In K Space With Location And Time With Taylor Series Solution(88)**

If $|\frac{1}{2\alpha k}.(\frac{1}{k}+ix-iwt)| < 1$ , when we use Taylor Series;

$$f(x,t,\alpha) = \int e^{-\alpha k^2} e^{ikx-iwkt} dk = -\frac{\frac{1}{2\alpha k}.(\frac{1}{k}+ix-iwt)}{1+\frac{1}{2\alpha k}.(\frac{1}{k}+ix-iwt)}.(\frac{1}{k}+ix-iwt)^{-2} \;.e^{-\alpha k^2}.e^{ikx-iwkt} + C \quad (88)$$

The position and time dependent wave equation is found.

## KAYNAKÇA

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