Comparison of GARCH and SVR-GARCH models: Example of gold return

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Abstract

Gold has been a precious resource for people on earth from the past to the present. It is used as both a value gain and jewelry, and is the focus of interest for people in terms of receiving attention and protecting its value. Especially recently, it has been the most favorite for investors due to its excess value increase and decrease which is constantly monitored. The study aimed to compare the predictive performance of the gold price return using the Support Vector Regression-GARCH hybrid models combined with the traditional volatility models. It has been examined whether the Support Vector Regression GARCH models would increase foresight performance. The study used data on the daily frequency between 01/01/2010–01/04/2023. Generalized Autoregressive Conditional Variable Variance, Glosten-Jaganthan-Runkle GARCH, Exponential GARCH and hybrid model Support Vector Regression -GARCH are utilized as prediction methods. For all methods, the gold series is divided into two groups as training and test data. The Root Mean Square Error values are compared as a model performance criterion. The RMSE values and graphics outputs have been concluded that the Support Vector Regression-GARCH model based on predicted linear, radial-based and polynomial kernel predicts more effectively than the GARCH models.

Keywords: Financial Market, GARCH Models, SVR-GARCH Model, Machine Learning, Time Series

JEL Codes: C22, C53, C58
1. INTRODUCTION

Since gold has been considered as an investment tool from the past to the present, it attracts the attention of individuals and institutions due to its exchangeability. Especially in recent times, the price value of gold has encouraged investors to follow gold more. Gold is constantly volatile likewise other financial instruments. This is due to the issues that occur within the country as well as in the globe. Therefore, it is important to model this volatility that financial markets have. This is because investors would like to make maximum profit from their investments. However, the high volatility of financial markets means that the risk is high. As a result of the high risk, investors want to have an insight into the financial assets they would invest in before making the actual investment.

The traditional methods (ARIMA, GARCH, etc.) are used to estimate volatility in their financial series. When the literature is examined, it is stated that recently machine learning algorithms are included in the predictions of the financial time series and making effective forecasts (Bildirici & Ersin 2012).

The econometric model commonly used for estimating volatility in financial markets is the ARCH model developed by Engle (1982). However, the ARCH model assumes that the impact of positive and negative news affecting the market on volatility is the same. Due to the diversity of the data and the differing problems, the ARCH model is developed by Bollerslev (1986) and is called the GARCH model. The ARCH and GARCH models assume that the variance effect of positive and negative shocks is the same. However, it seems that negative shocks representing bad news in the financial markets affect volatility more than positive shocks which represent good news. For this reason, the E-GARCH model, expressed as the exponential GARCH (E-GARCH), has been developed by Nelson (1991) to eliminate weaknesses ignored in the symmetrical models (Engle 1993: 75; Nelson 1991). One of the main shortcomings of the GARCH model is that this model does not consider the possible asymmetrical impact observed in the financial time series. Therefore, the GJR-GARCH model has been proposed by Glosten, Jagannathan and Runkle (1993), which takes into account the asymmetrical impact.

The study will estimate the GARCH, GJR-GARCH, E-GARCH and SVR-GARCH models and use the RMSE value determined as the model performance criterion. This study aims to examine whether the SVR-GARCH hybrid model is associated with higher performance compared to the GARCH, GJR-GARCH and E-GARCH models.

2. LITERATURE

When the literature is examined, Perez-Cruz et al. (2003) stated that the models predicted using SVR, in contrast to the GARCH models performed with the most probability estimation method applied to the time series in their studies, performed the best prediction.

Alberg et al. (2008) used GJR-GARCH, APARCH, E-GARCH models to estimate the return and conditional variance on the Tel Aviv Stock Exchange. The study utilized a variety of comparison criteria (i.e. MSE, MedSE, MAE and AMAPE and TIC) for comparison purposes and determined that the E-GARCH model with student-t distribution was the best predictor.

Ou and Wang (2010) used LS-SVM (Least Square Support Vector Machine), GARCH, E-GARCH, GJR-GARCH models to estimate the volatility of the ASEAN stock exchange. They stated that the LSSVM model provided more resistant and robust performance against volatility.

Jena and Goyari (2010) investigated the existence of a high volatility regime between 2005 and 2009, using the MS-ARCH model for oil and gold prices traded in the Indian market. The study reported that the high volatility was observed during the global financial crisis and the crisis is passed to a lower volatility regime after the crisis.

Bildirici and Ersin (2012) estimated BIST-100 index using the GARCH, SVR-GARCH and MLP (artificial neural networks)-GARCH models. It was concluded that SVR-GARCH and MLP-GARCH were better than the GARCH model.
Wang et al. (2013) compared error statistics for the model performance benchmark by performing Markov-switching (MSM), GARCH (1,1) and SVM-based Markov-switching (SVM-MSM) model forecasts for two different financial time series. The results showed that the best models were SVM-MSM, MSM and GARCH (1,1) respectively.

Gürsoy and Balaban (2014), using the BIST-100 index, estimated GARCH, E-GARCH, GJR-GARCH and SVR-GARCH models and stated that the best model was the SVR-GARCH model.

Karabacak et al. (2014) predicted volatility with the ARCH, GARCH, TARCH, E-GARCH and IGARCH models using the BIST-100 index return and gold return series in their study. TARCH stated that the best model for BIST 100 index return was the GARCH model for the gold return series.

Birau et al. (2015) used Bombay Stock Exchange Bank Index (BANK) used ARCH and GARCH models for volatility estimation. It was stated that the GARCH model predicted better than the ARCH model.

Katsiampa (2017) employed GARCH, EGARCH, TGARCH, APGARCH, C-GARCH, and AC-GARCH models for volatility modeling using Bitcoin data in his study. By comparing the AIC, SIC and HQ information criteria of the models, he determined that C-GARCH was the best model.

Cihangir and Uğurlu (2017) used GARCH, APARCH, TARCH and EGARCH estimation methods for the volatility of the gold price between 2010 and 2016 for their work. The study reported that APARCH was the most appropriate model.

Peng et al. (2018) examined the volatility of three different cryptocurrencies they identified. They applied GARCH, EGARCH, GJR-GARCH and SVR-GARCH models to the daily and hourly frequency data. The study reports that the SVR-GARCH model performs better than other models.

When the literature is examined econometrically, it is seen that gold prices and GARCH models are included in many studies. In this study, the hybrid model created by using the traditional methods and integrating with SVR, the recently widespread machine learning prediction algorithm, is created. For the performance evaluation of the applied prediction models, it is aimed to select the best prediction model by comparing the Root Mean Square Error (RMSE) values.

3. METHODS OF RESEARCH

Modeling and predicting volatility is very important in the financial markets. The high volatility states that the financial asset is risky. The correct estimate of the financial return volatility is crucial to assess investment risk.

Linear and nonlinear methods are used in time series. Linear models could be listed as ARIMA and GARCH. Support Vector Regressions and Artificial Neural Networks can be exemplified to nonlinear models. In this study, the hybrid GARCH model combined with SVR with the GARCH, GJR-GARCH, E-GARCH models will be estimated and the model with small error statistics will be determined.

The ARCH model is an autoregressive model developed by Engle (1982). ARCH is one of the most widely used models to model volatility in the market. It assumes that the variance value of the error terms is related to the previous period error values. It is possible to predict the variance of the series within a given time.

The ARCH model does not allow the conditional variance to change over time as a function of past errors which leaves the unconditional variance constant (Bolleslev 1986).
The ARCH model formulas are showed in Equation 1:

\[ y_t = \theta_0 + \theta_1 y_{t-1} + \cdots + \theta_n y_{n-1} + \epsilon_t \]
\[ \epsilon_t | _{t-1} N(0, \sigma_t^2) \]
\[ \sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_p \epsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 \]

ARCH model constraints exist: \( h_t \) and \( \epsilon_t \) must be positive. In other words, \( \alpha_0 > 0 \) other parameters \( \alpha_i \geq 0 \). The other constraint should be \( 0 \leq \beta_i \leq 1 \) (Engel 1982).

The GARCH model is developed by Bolleslev (1986) and Taylor (1986). The GARCH model estimates the conditional variance of the process variable based on its own delayed values. The error squares calculated in the mean equation gives information about volatility in past periods. When the literature is examined, it is seen that the GARCH model makes a more effective prediction than the ARCH model. The GARCH (1,1) model is the simplest but most powerful model of volatility (Engle 2001).

The GARCH model formula is showed in Equation 2:

\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \]  

As a GARCH model constraint is, \( \alpha_0 > 0 \), \( \alpha_i \geq 0 \), \( \beta_i \geq 0 \) and \( \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i < 1 \) (Bolleslev 1986).

The GJR-GARCH model is developed in 1993 by the Glosten, Jaganthan and Runkle. This model reacts to past negative and positive changes of the conditional variance. This model is recommended to determine the leverage effect in time series.

The GJR-GARCH model formula is showed in Equation 3:

\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \gamma_i \epsilon_{t-i}^2 I_{t-1} \sum_{i=1}^{p} \beta_j \sigma_{t-j} \]  

The \( I_{t-1} \) parameter in Equation 3 refers to unexpected news;

\[ I_{t-1} = \begin{cases} 1 & \text{if } \epsilon_{t-1} < 0, \ 	ext{bad news} \\ 0 & \text{if } \epsilon_{t-1} \geq 0, \ 	ext{good news} \end{cases} \]

is expressed.

\( \gamma \neq 0 \) states that it has an asymmetric effect. The case where \( \gamma > 0 \) indicates that the leverage effect is present. Leverage means bad news is more effective than good news (Engle & Sokalska 2012).

The constraints of the GJR-GARCH model are \( \alpha_0 > 0 \), \( \alpha_i > 0 \), \( \beta \geq 0 \), \( \gamma_l < 0 \) and \( \alpha_i + \gamma_i \geq 0 \) and (Wang and Wu 2012).

The E-GARCH model is proposed by Nelson (1991), who added the leverage effect to the model to enable the asymmetric effect to be seen. The E-GARCH formula is showed in Equation 4:

\[ \log(\sigma_t^2) = \alpha_0 + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{p} \alpha_i \left( \frac{\epsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{k=1}^{c} \gamma_k \frac{\epsilon_{t-k}}{\sigma_{t-k}} \]

The E-GARCH model responds asymmetrically to shock. Conditional variance is never negative due to logarithmic transformation in the model, it is always positive. The presence of \( \gamma_k < 0 \) in the model shows that the leverage effect exists (Çağlayan & Dayioğlu 2009). The advantage of the E-GARCH model allows unrestricted
SVR-GARCH is a powerful predictive method for predicting volatility and model risk. This is because the $h_t$ output from the GARCH model is used as input in the SVR. The kernel determined in SVR provides effectiveness according to the structure of the data.

Instead of replacing the maximum likelihood method with SVR to predict GARCH parameters, it is recommended to combine the SVR and GARCH models to predict volatility. First, the GARCH model is used to obtain $h_t$. Then, the nonlinear estimation is performed using the considering SVR model. In the equation is $Z_t = h_t^2 - h_t$. The linear GARCH model and the non-linear SVR model are combined to obtain estimates (Sun & Yu 2020).

In the SVR-GARCH model used to estimate volatility, the input vector is $x_t = [a^2, h_{t-1}]$, and the output variable is $h_t$. The SVR-GARCH structure is located below:

$$r_t = f(r_{t-1}) + a_t$$  \hspace{1cm} (5)

$$\tilde{h}_t = g(h_{t-1}, a_t^2)$$  \hspace{1cm} (6)

In Equation 6, $g$ is the decision function predicted by SVR for the mean equation. After the squared residues from the conditional mean estimate of SVR-GARCH, estimate the conditional variance equation given below:

In the mean equation, we will use 3 different kernels. The kernel functions are included in Table 1.

<table>
<thead>
<tr>
<th>Kernel Function</th>
<th>Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$K(x_i, x) = x_i^T x$</td>
</tr>
<tr>
<td>RBF</td>
<td>$K(x_i, x) = \exp(-\frac{1}{2\sigma^2}</td>
</tr>
<tr>
<td>Polynomial</td>
<td>$K(x_i, x) = (x_i^T x + 1)^d$, $d = 1, 2, ..., \infty$</td>
</tr>
</tbody>
</table>

The training set for determining the models and the test set are also used to evaluate the predictive performance of the models (Tay & Cao 2001). RMSE is used to evaluate predictive performance.

The RMSE value used to evaluate the effectiveness of the models and provides information concerning the active predictor with the least error. It is calculated as in Equation 7:

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^{n} \varepsilon_t^2}$$  \hspace{1cm} (7)
4. DATA AND APPLICATION

Observations of the gold variable used in the study are taken from the finance.yahoo website and closed values are used at the daily frequency between 01/01/2010 – 01/04/2023. The reason why this date range was chosen in the study includes the increase in gold prices in 2010 and the volatility that occurred in the global crisis in 2011. At the same time, the volatility in the price of gold was added to the model in the economic recession in the Covid-19 outbreak in 2019 and beyond. The polynomial kernel prediction process takes a lot of time. This data range has been studied as there is a timeout when working with more observations. The ARMA models are examined and ARMA (3,3) is determined before moving on to the GARCH models. The ARMA (3,3) prediction model is tested with ARCH-LM and rejected the basic hypothesis that the ARCH effect did not exist. As a result, it is concluded that it is appropriate to examine the GARCH models for the return series and the coefficient constraints of the models are examined.

The change in the gold price series variable over time is included in Figure 1.

![Gold Price Data](image)

**Figure 1. Gold Price Data**

When Figure 1 is examined, it is seen that the series has a trend. Due to the fact that the series has fracture and extreme values, as well as the changing variance, the series is converted in the return series to avoid the problem. In the case of a changing variance problem, the term error indicates that variances are related to the error terms of past periods. The formula for the return series of the index is reported in Equation 8:

\[ y_t = \ln \left( \frac{p_t}{p_{t-1}} \right) \ast 100 \]  

(8)

The Equation in 8 \( y_t \) refers to the return of gold, \( p_t \) refers to the price of gold. Figure 2 contains the graph of the gold return series.

![Gold Return Series](image)

**Figure 2. Gold Return Series**
When Figure 2 is examined, it is seen that the series fluctuates around zero and has volatility clusters. Large shocks follow large shocks, while small shocks follow small shocks. Descriptive statistics for the gold price and return series are included in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations</td>
<td>3424</td>
<td>3423</td>
</tr>
<tr>
<td>Mean</td>
<td>1460.444</td>
<td>0.01653</td>
</tr>
<tr>
<td>Median</td>
<td>1350.300</td>
<td>0.0269</td>
</tr>
<tr>
<td>Maximum</td>
<td>2069.400</td>
<td>5.7754</td>
</tr>
<tr>
<td>Minimum</td>
<td>1049.600</td>
<td>-9.8105</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>255.1708</td>
<td>1.0113</td>
</tr>
<tr>
<td>ADF</td>
<td>-1.4602</td>
<td>-59.8107</td>
</tr>
<tr>
<td>(0.553)</td>
<td>(0.000)*</td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>0.4265</td>
<td>-0.5942</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.7893</td>
<td>9.2772</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>312.9352</td>
<td>5821.326</td>
</tr>
<tr>
<td>(0.000)*</td>
<td>(0.000)*</td>
<td></td>
</tr>
</tbody>
</table>

Note: * indicates the rejection of the null hypothesis that the series is unit root according to 5% for the ADF test. Indicate the rejection of the null hypothesis that the distribution is normal according to 5% for the Jarque-Bera test.

The study is carried out with 3423 observations taking the return of the gold price, which is 3424 observations used in the study. ADF unit root test is applied to the gold price data and return data. While the price series has unit root, it is seen that the return series becomes stationary when the difference is taken. The Jarque-Bera test does not appear to have normal distribution for both the price series and the return series. 252 observations of 3423 observations are determined as test data, while the remaining observations are determined as data set training. As a model performance criterion, the RMSE error statistical criterion is taken into account.

Figure 3 shows the graphical output for ARCH model estimate.

![Figure 3](image.png)

**Figure 3.** Comparison of the ARCH Model Volatility Estimate with the Actual Values

Figure 3 reveals that the ARCH model estimate does not exactly match the actual value. The forecast shows higher volatility than actual values.
Figure 4 shows the graphical output for the GARCH model estimate.

Figure 4. Comparison of the GARCH Model Volatility Estimate with the Actual Values

According to Figure 4, it is seen that the GARCH estimate does not overlap with the actual values and the estimate is insufficient.

Figure 5 shows the graphic output of the GJR-GARCH model estimate.

Figure 5. Comparison of the GJR-GARCH Model Volatility Estimate with the Actual Values

Once Figure 5 is taken into consideration, it could be seen that the actual volatility values and the GJR-GARCH model estimate do not overlap. This estimate appears to be insufficient to capture endpoints in volatility.

Figure 6 shows the graphic output of the E-GARCH model.

Figure 6. Comparison of E-GARCH Model Volatility Estimate with the Actual Values

Figure 6 suggests that E-GARCH cannot capture the actual volatility values with the estimate. This estimate meth-
od appears to have failed to catch endpoints.

Figure 7 shows the estimate graph for the linear SVR-GARCH model.

![Figure 7](image)

**Figure 7.** Comparison of Linear SVR-GARCH Model Volatility Prediction with the Actual Values

When Figure 7 is examined, it is seen that the linear SVR-GARCH model estimate shows very close estimate to actual values.

Figure 8 shows the estimate output of the RBF SVR-GARCH model.

![Figure 8](image)

**Figure 8.** Comparison of RBF SVR-GARCH Model Volatility Estimate with the Actual Values

When Figure 8 is examined, it is seen that the RBF SVR-GARCH estimate overlaps with real value and is good at capturing endpoints. The forecast appears to have performed successfully in capturing the endpoints.

Figure 9 shows the graphic output for the Polynomial SVR-GARCH model estimate.

![Figure 9](image)

**Figure 9.** Comparison of Polynomial SVR-GARCH Model Volatility Prediction with the Actual Values
When Figure 9 is examined, it is seen that the Polynomial SVR-GARCH estimate not good at capturing real values and makes insufficient estimate.

The graphs of the prediction models can be interpreted, but the results we can rely on in the study are the model performance criteria. In this study, RMSE will be considered as the model performance criterion. Table 3 contains RMSE values as a model performance measure.

<table>
<thead>
<tr>
<th>Prediction Method</th>
<th>RMSE</th>
<th>Model Performance Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>0.0913</td>
<td>7</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.0879</td>
<td>4</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>0.0880</td>
<td>5</td>
</tr>
<tr>
<td>E-GARCH</td>
<td>0.0904</td>
<td>6</td>
</tr>
<tr>
<td>Linear SVR-GARCH</td>
<td>0.0008</td>
<td>2</td>
</tr>
<tr>
<td>RBF SVR-GARCH</td>
<td>0.0007</td>
<td>1</td>
</tr>
<tr>
<td>Polinomial SVR-GARCH</td>
<td>0.0018</td>
<td>3</td>
</tr>
</tbody>
</table>

The RMSE values in Table 3 show that the models with the small statistics are hybrid models. The RMSE value of ARCH and GARCH models is 0.0913 and 0.0879, respectively. These models ignore the asymmetrical effect. GJR-GARCH has been proposed to solve the asymmetry problem in the financial series, and the RMSE value is 0.0880. GARCH and E-GARCH models feature short memory. This feature does not comply with long-term estimates. For this reason, the training set is kept smaller than the test set and the RMSE value is 0.0904. The SVR-GARCH model shows that the predictive performance is better, with less error statistics than GARCH models. SVR-GARCH models appear to give the RBF kernel with the smallest error statistics value 0.0007 due to the compatibility of the kernel to the data set.

5. CONCLUSION

Gold is the precious metal that people have used for centuries as a means of exchange and investment. Investors are curious about the future movements of gold prices because of being a valuable mine. Investors want to know the volatility in order not to suffer losses and to make a maximum profit in their investments. Even countries try to understand the volatility in financial prices when making a decision to invest in each other. For this reason, it is important to determine the appropriate time series model for researchers.

In this study, the gold price series is obtained through yahoofinance.com and daily closing values are used. Traditional estimation methods of GARCH and hybrid SVR-GARCH Linear, RBF, Polinom kernels are used to take into account the nonlinear structure with time-varying volatility. When the graphs in the estimates are examined, the prediction effectiveness of GARCH models appears to be low. SVR-GARCH models, it seems more effective in predictions and closer to catching volatility. Considering the RMSE values determined as the model performance metric, it appears that the graphics are accurately related to the output and hybrid models have less error statistics. As a result of the study, it appears that the minimum error statistics value is belong to the hybrid model RBF kernel SVR-GARCH.

According to this study, considering the extreme volatility of the financial time series and its non-linear structure, it is concluded that hybrid models will be more accurate to prefer than traditional methods. In the study, ARCH, GARCH, GJR-GARCH, E-GARCH and SVR-GARCH models are predicted and compared. The SVR-GARCH hybrid model is created in three different prediction models as Linear, RBF and Polynomial kernels. When RMSE statistics and graphics are examined, it is seen that SVR-GARCH models have the best performance which is estimated by three different kernels. It is obvious that SVR-GARCH captures the volatility clusters in the graphics better. Therefore, it is suggested that SVR-GARCH hybrid models can be used in financial time series estimates.
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Submission Declaration Statement

We confirm that this work is original and has not been published elsewhere, nor is it currently under consideration for publication elsewhere.
REFERENCES


